

# A traffic congestion control method in the cyber physical systems\*

Di-Hua Sun, Tong Zhou, Min Zhao, Wei-Ning Liu, Zhiyong Yang, Geng Zhang and Hui Liu

**Abstract**— Congestion control is a process to achieve optimal decision-making and effective control of vehicles through the interaction of vehicular perception and control, and it shows the typical characteristic of cyber physical systems. Based on the physical-world model of Konishi et al., a new control method to suppress the traffic jam is proposed in the feedback cyber physical systems under the open boundary, which considers the effect of safe headway on the traffic system from the perspective of tight conjoining between the transportation cyber system and the transportation physical system. By using the Hurwitz stability criteria and the  $H^\infty$  norm theory, the condition under which the traffic jam can be well suppressed is analyzed. Comparison between our method and the previous methods is carried out. The simulation results show that the temporal behavior obtained by our method is better than those of the previous control methods, and our control method could be well applied to traffic congestion control in cyber physical systems. The simulation results are consistent with the theoretical analysis.

## I. INTRODUCTION

Congestion control is a process that physical-world sensors sense the physical state information of vehicles, then transmit the information to cyber-world computing units for

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analysis, process and decision-making, then send the control information to actuator to adjust the running state of vehicles in the physical-world, and make traffic flow(vehicles) rapidly recover steady state, and the control effect in physical-world will again transmit to the cyber-world for further optimal congestion control. It can be found that congestion control is the process of interaction between the transportation physical world and the transportation cyber world, therefore, it shows the typical characteristic of cyber physical systems which is a close-loop system. Researchers have been studying on congestion control method in the traffic close-loop system, but the control effect is not too ideal. Our work aims at achieving the optimal stability control of traffic flow in feedback cyber physical system.

Based on the previous work, the present paper proposes a new feedback cyber physical system with consideration of the safe headway effect. The contributions of the paper are as follows: In Sec.2, a new feedback cyber physical system is introduced; In Sec.3, the stability analysis of the system is given; In Sec.4, simulations results are performed to verify the theoretical analysis results. The concluding section summarizes the paper.

## II. RELATED WORKS

A cyber physical system is an integration of physical components and cyber-world components through computation, communication and control technologies for generating intelligent behavior[1]. The physical components are used as sensors and to control entities in the physical world, and cyber-world components has computing and communication capabilities for data analysis and process, and information interaction. CPS fully embodies the depth fusion through the interaction and feedback between the cyber world and physical world.

In 1995, special emphasis is placed on CPS by NSF, and a number of international conferences on CPS are held for its theories and key technologies. Especially since 2008, CPS week is held once a year in United States. CPS is viewed as key research direction in NSFC, “973” and “863”. Wang et al.[2] design the architecture of transprotation CPS. Wen et al.[3] summarize the basic concepts and nature of CPS, and gives a systemic review of recent progress on key technologies respectively. Deshmukh et al.[4] estimated state over a lossy network in spatially distributed cyber physical systems. Sun et al. [1] discuss the concept, architecture, key technologies and challenges of transportation CPS. The results mentioned above can support theoretical basic for congestion control.

At present, the theoretical research of congestion control is based on the traffic flow model, which is to build a physical-world model to describe the interaction relationship among vehicles in the physical-world [5]. The most

well-known physical-world model is the optimal velocity (OV) model [6], which has successfully revealed the dynamical evolution process of traffic flow in a simple way. In the aspect of decision-making and control of vehicles, according to the discrete version of the OV model (the coupled map car-following model)[7], Konishi et al.[8] proposed a decentralized delayed-feedback control method to control the driving behavior of vehicles for reducing the influence of disturbance on traffic flow, and suppressing the jam in 1999. In 2006, Zhao and Gao [9] presented another simple strategy for congested state in the traffic system. In 2007, Han et al. [10] presented a modified CM car-following model, in which the control signal was proportional to the velocity difference between the successive vehicles of an arbitrary number of preceding cars. After that, many important achievements related to the control method[12,13]have been obtained recently. Previous studies show that control method can improve the stability of traffic flow for suppressing traffic jam, therefore, which can enhance the fusion degree of the transportation physical world and the transportation physical world.

These control methods mentioned above in feedback cyber physical system just using the relative speed with the consideration of the successive vehicles of arbitrary number of preceding vehicles of preceding vehicles as a feedback control signal for suppressing traffic jam, however, they cannot be used to study the influence of the safe headway effect on the interaction relationship among vehicles. In fact, when a driver sense the headway of his car is smaller than the safe headway, the driver controls immediately the speed of his car for keeping a safe distance with his leading car to avoid collision for suppressing traffic jam. Therefore, considering the safe headway effect can also enhance the control performance of the physical world, and make traffic flow become stable to suppress traffic jam. In view of the reasons given above, the present paper proposes a new feedback control method by considering the safe headway effect in the physical-world model.

### III. THE PHYSICAL MODEL OF TRAFFIC FLOW

Our investigations are based on the CM car-following model[7]. By using the physical-world vehicle detector to sense the running state of vehicles, for example, the leading vehicle's the position  $x_0(n)$  and velocity  $v_0$  at time  $t = nT$ , the initial position of leading vehicle  $x_0(0)$ , and the position  $x_i(n)$  and speed  $v_i(n)$  of the  $i$ th vehicle at time  $t = nT$ , then transmit the information to the cyber-world to build a physical-world model for describing the interaction relationship among vehicles.

The leading vehicle is described as follows:

$$x_0(n+1) = v_0 T + x_0(n) \quad (1)$$

where  $T$  is the sampling interval.

The following vehicle  $i$  is described as

$$x_i(n+1) = v_i(n)T + x_i(n) \quad (i = 1, 2, K, N) \quad (2)$$

where  $N$  is the total number of vehicles. The speed of the following vehicles is governed by

$$v_i(n+1) = a_i \left( V_i^{OP}(y_i(n)) - v_i(n) \right) + v_i(n) \quad (3)$$

where  $a_i$  is the sensitivity of the  $i$ th vehicle driver,  $V_i^{OP}(y_i(n))$  is the OV function, which depends on the headway distances  $y_i(n)$ :

$$y_i(n) = x_{i-1}(n) - x_i(n) \quad (4)$$

The OV function is given by[6]

$$V_i^{OP}(y_i(n)) = \frac{v_i^{\max}}{2} \left( 1 + \frac{h}{y_i(n)} \right) + \bar{H}_{sat} \left( \frac{y_i(n) - h}{z} \right) \quad (5)$$

where  $v_i^{\max}$  is the maximum speed,  $h$  is the safe headway and  $z$  is a parameter, the saturation function  $\bar{H}_{sat}(\cdot)$  is described as

$$\bar{H}_{sat} = \begin{cases} +1, & \text{if } r > +1; \\ r, & \text{if } -1 \leq r \leq +1; \\ -1, & \text{if } r < -1. \end{cases} \quad (6)$$

To guarantee the existence of traffic flow, it is assumed that the velocity of the leading vehicle is less than the maximum velocity of all the following vehicles.

To avoid the collisions and backward motions, all the vehicles adopt the full braking action:

$$\text{if } y_i(n) < y_i^{\min} \quad (7)$$

then

$$x_i(n+1) = x_i(n), v_i(n+1) = 0 \quad (8)$$

This action implies that the  $i$ th vehicle stops suddenly when the headway  $y_i(n)$  is less than a minimum headway.

The physical-world model of the  $i$ th vehicle can be given as:

$$\begin{aligned} v_i(n+1) &= a_i \left( V_i^{OP}(y_i(n)) - v_i(n) \right) + v_i(n) \\ y_i(n+1) &= v_{i-1} T - v_i(n) T + y_i(n) \end{aligned} \quad (9)$$

The steady state of the physical system(9) is

$$[v_i^*, y_i^*]^T = [v_0, \frac{v_0}{r_i} - \frac{z_i}{2} + h_i]^T \quad (10)$$

where  $r_i = v_i^{\max} / z_i$ . According to control theory, the system(9) to be stable as follows:

$$0 < r_i < \frac{1}{T}, \quad 0 < a_i < \frac{4}{T(2 - r_i T)} \quad (11)$$

$$\frac{8 + a_i T(a_i T - 8)}{a_i T^2(a_i T - 6)} < r_i < \frac{a_i}{2 + a_i T} \quad (12)$$

We confirm that traffic congestion never occurs when all the vehicles satisfy the conditions (11) and (12). In the next section, we will propose a new control method to form a feedback cyber physical model for suppressing traffic jam.

### IV. CYBER PHYSICAL MODEL OF TRAFFIC FLOW

In the real traffic, when the headway of the considered car is less than the safe headway, the cyber system has to adjust the velocity of his car based on the information

about the gap between the safe headway and the headway of the considered car, the velocity difference between the considered vehicle and its front one to avoid collision, and the safe headway is the first factor to be considered. In turn, when the headway is larger than the safe headway, it's the cyber system according to the information adjusts the velocity about velocity difference to keep up with the leading one, without considering the safe headway effect. Therefore, a new feedback control signal  $u_i(n)$  is designated as follows:

$$u_i(n) = 0.85Vv_i(n) - kH(y_i(n) - h)(h - y_i(n)) \quad (13)$$

where  $k$  is the feedback gain. The function  $H(\cdot)$  is described as follows:

$$H(y_i(n) - h) = \begin{cases} 0 & y_i(n) - h > 0 \\ 1 & y_i(n) - h \leq 0 \end{cases} \quad (14)$$

For  $y_i(n) - h > 0$ , our feedback control signal  $u_i(n)$  can be reduced to the control signal of Zhao et al. as follows:[7]

$$u_i(n) = 0.85Vv_i(n) \quad (15)$$

For  $y_i(n) - h \leq 0$ , our control signal is designed as follows:

$$u_i(n) = 0.85Vv_i(n) - k(h - y_i(n)) \quad (16)$$

The control signal term is added to the physical-word model(9), and the feedback cyber physical system is obtained as follows:

$$\begin{aligned} v_i(n+1) &= a_i v_i(n) + u_i(n) + v_i(n) \\ y_i(n+1) &= v_i(n)T + y_i(n) \end{aligned} \quad (17)$$

In the control signal  $u_i(n)$ , we take the velocity difference between the  $v_{i-1}(n)$  in front and  $v_i(n)$  into account as well as the difference between the  $y_i(n)$  and  $h$  in time  $t = nT$ . The closed-loop system, which consisting of Eqs. (2), (4) and (17), has the same steady state as that of Eq. (10). Around the steady state, the controlled system(17) can be described by

$$\begin{aligned} \frac{dv_i(n+1)}{dy_i(n+1)} &= \frac{0.15 - a_i T}{-T} \frac{a_i r_i T + k}{1} \frac{dv_i(n)}{dy_i(n)} \\ &+ \frac{0.85V}{T} \frac{dv_{i-1}(n)}{dy_i(n)} \end{aligned} \quad (18)$$

The transfer function  $G_i(z)$  from  $dv_{i-1}(n)$  to  $dv_i(n)$  is described by

$$\begin{aligned} G_i(z) &= (1 - a_i T) \frac{0.15 - a_i T - k - a_i r_i T}{T(z - 1)} + \frac{0.85V}{T} \\ &= \frac{0.85V(z - 1) + pa_i r_i T^2 + kT}{P_i(z)} \end{aligned} \quad (19)$$

where  $P_i(z) = z^2 + a_i z + b_i$ ,  $a_i = a_i T - 1.15$ ,  $b_i = 0.15 - a_i T + kT + a_i r_i T^2$ .

In order to suppress the traffic jams in the controlled system (17), the feedback gain  $k$  is designed such that

- (i)  $P_i(z)$  is stable,
- (ii)  $\max_{|z|=1} |G_i(z)| \leq 1$  ( $i = 1, 2, \dots, N$ ).

**Theorem 1** There is no traffic jam in the controlled system (17),

- (i) if  $0 < a_i r_i T^2 < \min\{2, 2a_i T - 2.3\}$  then the feedback gain  $k$  is set as  $\max\{\frac{0.15 - a_i T + a_i r_i T^2}{T}, R_1\} < k < \frac{2.3 - 2a_i T + a_i r_i T^2}{2T}$  (20)

Where  $R_1 = \frac{-1.15 + 2.7a_i T - 3a_i r_i T^2 + a_i^2 T^2 - a_i^2 r_i T^3}{3T + a_i T^2}$  (22)

- (ii) if  $\max\{0, 2a_i T - 2.3\} < a_i r_i T^2 < 2$  then the feedback gain  $k$  is set as  $\frac{0.15 - a_i T + a_i r_i T^2}{T} < k < \min\{\frac{2.3 - 2a_i T + a_i r_i T^2}{2T}, R_1\}$  (24)

- (iii) if  $0 < a_i r_i T^2 < \min\{4, 2a_i T + 1.7\}$  then the feedback gain  $k$  is set as  $\max\{\frac{-a_i T - 0.85 + a_i r_i T^2}{T}, R_2\} < k < \min\{\frac{2.3 - 2a_i T + a_i r_i T^2}{2T}, \frac{0.15 - a_i T + a_i r_i T^2}{2T}\}$  (26)

Where  $R_2 = \frac{-0.55 - 1.3a_i T + a_i r_i T^2 + a_i^2 T^2 - a_i^2 r_i T^3}{T - a_i T^2}$  (27)

- (iv) if  $2a_i T + 1.7 < a_i r_i T^2 < 4$  then the feedback gain  $k$  is set as  $\frac{-a_i T - 0.85 + a_i r_i T^2}{T} < k < \min\{\frac{2.3 - 2a_i T + a_i r_i T^2}{2T}, R_2\}$  (29)

- (v) if

$$0 < a_i r_i T^2 < 2, k = \frac{0.15 - a_i T + a_i r_i T^2}{T} \quad (30)$$

then the feedback gain  $k$  is set as

$$\frac{0.15 - a_i T + a_i r_i T^2}{T} < k < \frac{2.3 - 2a_i T + a_i r_i T^2}{2T} \quad (31)$$

This theorem provides us with the ability to design the feedback gain  $k$  to suppress the traffic jam in our cyber physical model. From the theorem, we can derive stability conditions to design the feedback gain, and then the traffic jam never occurs in the controlled vehicle group.

## V. SIMULATION RESULTS

The aim of this section is to investigate whether the result of the theoretical analysis in Section 3 holds. Our simulations are based on the physical-world model under open boundary conditions. The parameters of the system are set as follows [7]:

$$\begin{aligned} h &= 25.0 \text{ m}, z = 23.3 \text{ m}, v^{\max} = 33.6 \text{ m/s}, \\ a_i &= 2.0 \text{ s}^{-1}, v_0 = 20 \text{ m/s}, T = 0.1 \text{ s}, \\ y^{\min} &= 7.02 \text{ m}, r_i = v^{\max} / z \gg 1.44 \text{ s}^{-1}. \end{aligned}$$

It is assumed that all vehicles have the same parameters. The initial conditions are the steady state of the system. The initial positions and speeds are set to be

$$x_i(0) = \sum_{j=i+1}^N \mathbf{a} \quad y_j(0) = y_j^*, v_i(0) = v_i^*, i = 1, 2, \dots, N \quad (32)$$

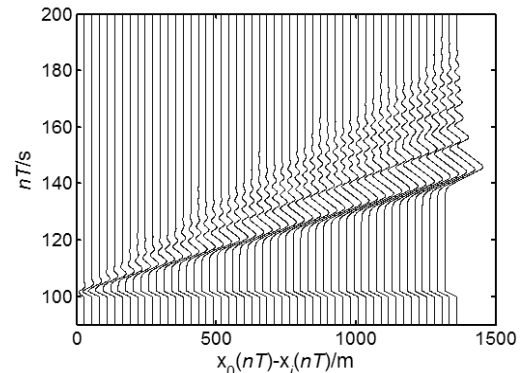
Consider a situation where the leading vehicle stops suddenly for a short time [7]:  $v_0(n) = 0, nT=100-102 \text{ s}$ .

The external disturbance is used to examine the control effect of our model on traffic jam. Now, let us set the feedback gain  $k$  in our method. By calculation, it is founded that these parameters fit three conditions(i), (iv), (v). Substituting the parameter values into Eq.(21), Eq.(29) and Eq.(31), then  $0 < k < 0.1013$  is obtained. In the method of Konishi et al. the procedure to design four feedback gains can be found in Ref. [7].

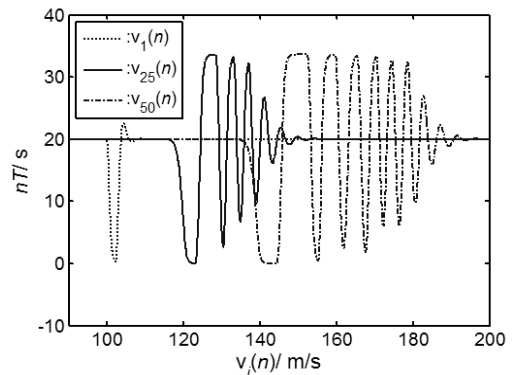
Figures 1, 2, 3 and 4 show the space-time plot of traffic flow and the variation of velocities of the first, the 25th and the 50th vehicle in the physical-world model, the physical-world model controlled by Konishi et al., Zhao et al., and our method respectively.

It can be seen that the stop disturbance propagates backwards in the physical-world model. From Figs. 2(a), 3(a) and 4(a), we observe that there is no oscillating behavior and traffic jam when control signals are introduced in the feedback cyber physical system. All the vehicles are running smoothly without full braking in the three feedback controlled systems. However, it should be noted that the headway fluctuation is gentle in Fig. 4(a) compared with those in Figs. 2(a) and 3(a). In Fig. 1(b), it is found that when traffic flow is disturbed, the three vehicles velocity perturbations will be amplified with time and finally evolve into stop-and-go state. From Figs. 2(b), 3(b) and 4(b), we can see that the vehicles are moving smoothly as the amplitude of speed fluctuation decreases with the increase of vehicle number  $i$ . However, it should be noted that the amplitude of speed fluctuation is the smallest in our controlled system, compared with the physical-world model

controlled by Konishi et al. and Zhao et al. The simulation results indicate that three cyber physical systems can be used to suppress the traffic jam, and the physical-world model exhibits better temporal behavior in our feedback cyber physical system.

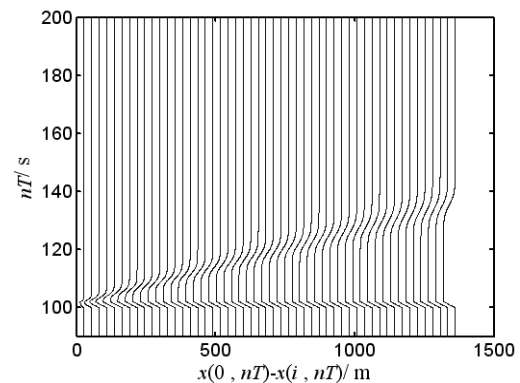


(a) Space-time plot of the physical-world model.

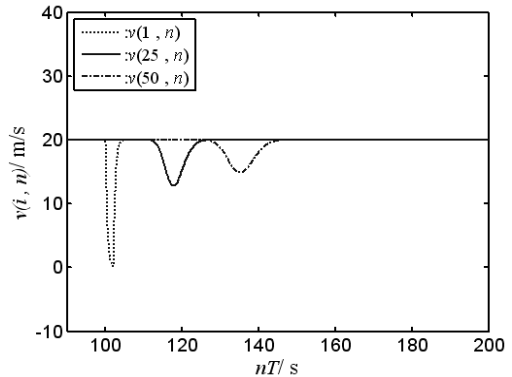


(b) Temporal velocity behavior of the first, 25th and 50th vehicles.

Figure 1. Numerical simulations in the physical-world model.

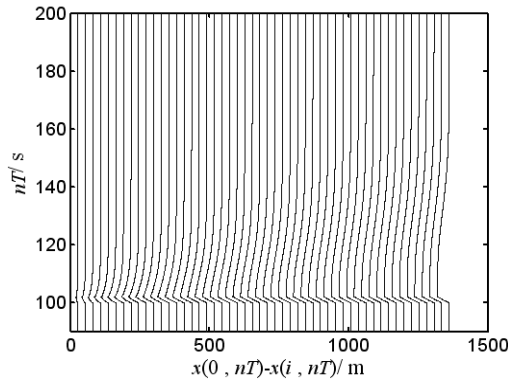


(a) Space-time plot of the CPS controlled by Konishi et al. method.

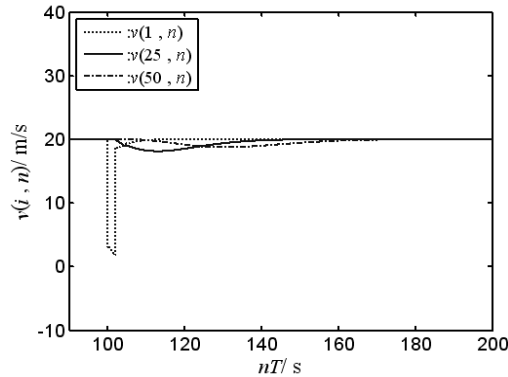


(b) Temporal velocity behavior of the first, 25th and 50th vehicles.

Figure 2. Numerical simulations in the CPS controlled by Konishi et al. method.

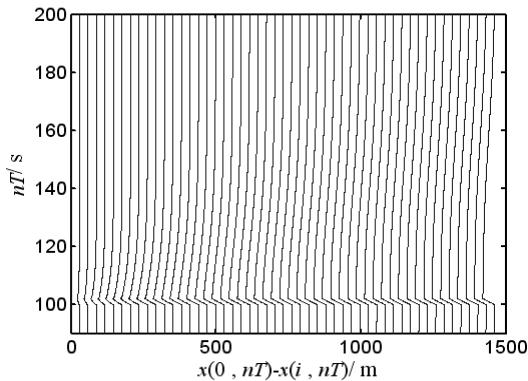


(a) Space-time plot of the CPS controlled by Zhao et al. method.

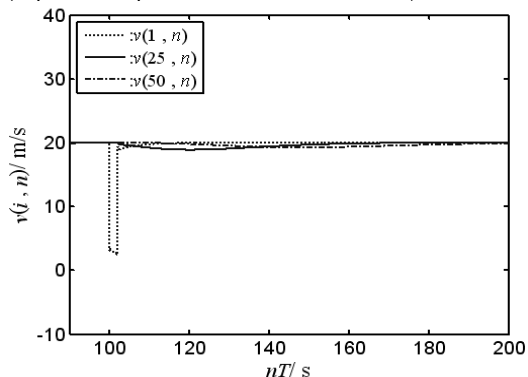


(b) Temporal velocity behavior of the first, 25th and 50th vehicles.

Figure 3. Numerical simulations in the CPS controlled by Zhao et al. method.



(a) Space-time plot of the CPS controlled by our method.



(b) Temporal velocity behavior of the first, 25th and 50th vehicles.

Figure 4. Numerical simulations in the CPS controlled by our method.

## VI. CONCLUSION

In the paper, a new control method for the suppression of the traffic jam is proposed based on the pioneering work of Konishi et al. [6], which can more constitutionally react to cyber-physical characteristics of congestion control. The control effect of our feedback cyber physical system is investigated, and compared with the previous work under same parameters, three advantages of our method as follows: (i) Traffic jam can be suppressed by introducing the feedback control signal. (ii) The stability of a traffic system becomes better by considering the influence of safe headway. (iii) The numerical simulations are in good agreement with the theoretical analysis. In future, we will adjust the parameter of our control model with empirical data for wide application in real traffic.

## REFERENCES

- [1] D. H. Sun, Y. F. Li, W. N. Liu, M. Zhao and X. Y. Liao, "Research summary on transportation cyber physical systems and the challenging technologies," *China Journal of Highway and Transport*, vol. 26, no.1, pp. 144-156, Jan. 2013.
- [2] Y. D. Wang, G. Z. Tan, Y. Wang and Y. Yin, "Perceptual control architecture for cyber-physical systems in traffic incident management," *Journal of Systems Architecture*, vol. 58, no. 10, pp.398-411, Nov. 2012.
- [3] J. R. Wen, M. Q. Wu and J. F. Su, "Cyber-physical system," *Acta Automatica Sinica*, vol.38, no.4, pp.507-518, Apr. 2012.
- [4] S. Deshmukh, B. Natarajan and A. Pahwa, "State estimation over a lossy network in spatially distributed cyber-physical systems," *IEEE Transactions on Signal Processing*, vol. 62, no. 15, pp. 3911-3923, Aug. 2014.
- [5] L. Yu, Z. K. Shi and B. C. Zhou, "Kink-antikink density wave of an extended car-following model in a cooperative driving system," *Commun. Nonlinear Sci. Numer. Simul.*, vol.13, no.10, pp. 2167-2176, 2008.
- [6] M. Bando, K. Hasebe, A. Nakayama, A. Shibata and Y. Sugiyama, "Dynamical model of traffic congestion and numerical simulation," *Phys. Rev. E*, vol.51, no.2, pp.1035-42, 1995.
- [7] S. Yukawa and M. Kikuchi, "Coupled-Map modeling of one-dimensional traffic flow," *J. Phys. Soc. Jpn*, vol.64, no.1, pp.35-38, 1995.
- [8] K. Konishi, H. Kokame and K. Hirata, "Coupled map car-following model and its delayed-feedback control," *Phys. Rev. E*, vol.60, no.4, pp.4000-4007, 1999.
- [9] X. M. Zhao and Z. Y. Gao, "A control method for congested traffic induced by bottlenecks in the coupled map car-following model," *Physica A*, vol. 366, no.8, pp. 513-522, 2006.
- [10] X. L. Han, C. Y. Jiang, H. X. Ge and S. Q. Dai, "A modified coupled map car-following model based on application of intelligent transportation system and control of traffic congestion," *Acta Physica Sinica*, vol. 56, pp. 4383-4392, 2007.
- [11] H. X. Ge, X. P. Meng, J. Ma and S. M. Lo, "An improved car-following model considering influence of other factors on traffic jam," *Phys. Lett. A*, vol. 377, no.1, pp. 9-12, 2012.
- [12] R. J. Cheng, X. L. Han, S. M. Lo and H. X. Ge, "A control method applied to mixed traffic flow for the coupled-map car-following model," *Chin. Phys. B*, vol. 23, no.3, pp. 030507, 2014.